FREE-CONVECTION HEAT TRANSFER IN A HORIZONTAL CYLINDRICAL LAYER WITH VARIABLE HEAT SOURCE LOCATION

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The method and results of an experimental investigation of free-convective heat transfer with variable heat source position are reported.

As noted in [1], the obtaining and generalizing of test data on heat transfer from bodies in closed volumes is one of the major problems involved in application of the theory of the thermal regime of radio, electronic, and measuring equipment.

It was established experimentally in [3] that the surface temperature of a heat-generating element depends on its location in a closed space, and the question arises of a more detailed investigation of the qualitative and quantitative aspects of the problem. This paper deals with the technique of this kind of investigation and its results.

The basic elements of the experimental setup (Fig. 1) are a thin-walled tube 1 made of polished nickel-plated foil (thickness Δ = 0.25 mm), an electric heater 2 in the form of a thin-walled, polished, nickel-plated tube with an internal nichrome coil wound at constant pitch on a ceramic rod, and fluoroplastic thin-walled caps 3 of the tube with a groove for guiding the heater during its displacement, these simultaneously providing insulation at the end of the tube and the heater. Tube 1 was fastened with clips to the table of a drilling machine, so that it could be traversed with an accuracy of 0.05 mm, while the heater was rigidly "suspended" from a wall bracket by means of two chrome-molybdenum tubes of diameter 8 mm (brought out through gasketed apertures in tube 1).

The heater power supply was a stabilized VS-12 rectifier. The current was determined with the aid of an R-330 potentiometer from the voltage drop in a standard R-321 0.1 m Class 0.01 resistor, while the voltage drop in the heater was measured with a Class 0.1 M-502 voltmeter.

The heater surface temperature was measured with 12 copper-constantan thermocouples soldered into the tube wall. The thermal wires, 0.1 mm in diameter, were glued to the inside wall of the heater tube and brought out through the chrome-molybdenum tubes. The heater leads were also located in them. The central transverse section of the heater had 4 thermocouples, and there were two thermocouples in each of four other sections, 250 and 500 mm to right and left of the center. For measuring its temperature, the surface of tube 1 had 20 thermocouples, of which 12 were in the central cross section (at equal intervals around the periphery), while there were four in each of the sections 250 mm to right and left of center. The thermocouple wires were stretched along generators of the cylinder up to the end pieces, where they ran together into common bundles.

All the thermocouple leads were connected through the switch to the measuring system, which consisted of a low-impedance R330 potentiometer 4, a standard element 5 of Class II accuracy, a M-195 null galvanometer 6, and a dc battery 7.

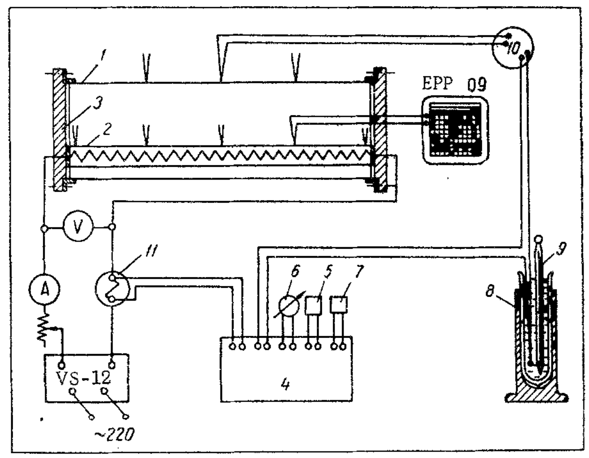

Fig. 1. Diagram of the experimental setup: 1) external tube; 2) heater; 3) end insulators; 4) low-impedance potentiometer; 5) standard; 6) null galvanometer; 7) dc battery; 8) cold-junction thermostat; 9) reference thermometer; 10) switch; 11) standard resistor; V—voltmeter; A—ammeter.

The common cold junction for all the thermocouples was located in a Dewar flask with glycerin 8, whose temperature was measured with a reference thermometer 9 with 0.1° C subdivisions.

For all variations of the heater diameter (D_h = 16; 28; 40 mm) and of the outside tube (D_2 = 80 and 160 mm), the length L was 1000 mm. The tests spanned a range of eccentricity e from 0 to $\pm[D_2/2 - (D_h/2 + 2)]$ mm (which corresponds to a minimum slit between the tube and the heater of 2 mm), for ratios of D_2/D_h = = 2; 4; 5.7; 10 at three power conditions, corresponding to a temperature difference between the heater surface and the tube wall from about 40°–45° to 150°–160° C. The convective medium was air.

All the measurements were conducted under strictly steady-state conditions, this being monitored with the help of an EPP-09 potentiometric recorder, to which one of the heater thermocouples was connected.

Since the "infinite length" condition of the tube-heater system was comparatively well satisfied, the data of the "working section" must be taken as calculation quantities. Thus the "working section" turned

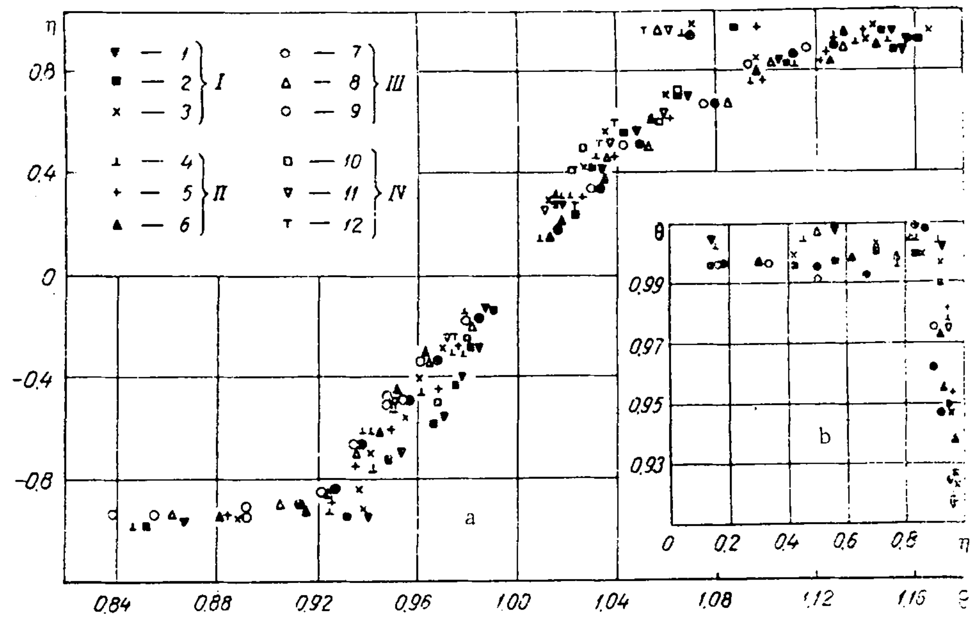

Fig. 2. Experimental data on variation of dimensionless temperature for a) vertical and b) horizontal displacement: I) for the pair of tubes with D_2 = 160 mm, D_h = 16 mm, with I = 0.165 A (1), 0.22 (2), 0.28 (3); II) the same with D_2 = 160 mm, D_h = 28 mm and with I = 0.16 A (4), 0.23 (5), 0.3 (6); III) D_2 = 160 mm, D_h = 40 mm, I = = 0.19 A (7), 0.25 (8), 0.3 (9); IV) D_2 = 80 mm, D_h = 40 mm, I = 0.19 A (10), 0.25 (11), 0.3 (12).

out to be the middle part of the tube and a length l_{ws} = = 500 mm of the heater. The end effect here was not appreciable, since the temperature difference between the thermocouples adjacent to the central thermocouples was at most 0.5% of the temperature in the central heater section. Therefore the average thermocouple readings over a length of 0.5 m were taken as the heater and outer tube calculation temperatures, while the power in the working section was assumed to be $Q_{ws} = (l_{ws}/L)Q = (500/1000)Q$.

The data obtained, reduced to the dimensionless form $[\theta = (t_1 - t_2)_e/(t_1 - t_2)_0,\ \eta = \pm e/(R - r)]$, are shown graphically for vertical (Fig. 2a) and horizontal (Fig. 2b) displacement of the heater.

Figure 2a shows a clear dependence of relative temperature difference on relative vertical eccentricity, which is generalized for all the tests, with sufficient accuracy for technical purposes. At maximum positive and negative vertical eccentricities, temperature discontinuities are observed in the direction of lower temperature, which may be fully accounted for because of the increased heat transfer in the thin boundary layer.

Figure 2b allows us to assert that the eccentricity in the horizontal plane has no influence on the thermal conditions of the surface of the heat-generating element. A similar temperature discontinuity is observed at the maximum eccentricities.

The results of the present investigation (with the dependence of Fig. 2 being expressed by an equation) enable more accurate calculation of the surface temperature of the heat source when it is displaced in a closed volume. It is evident that this calculation is based on reliable heat transfer data in horizontal cylindrical layers with a centrally located heater.

It is known [4, 5] that the amount of heat transmitted through a cylindrical layer of liquid or gas under natural convection is given by the expression

$$Q = \mathrm{Nu}'\, 2\pi l\, (t_1 - t_2)\, \lambda_f / \ln (D_2/D_h).$$

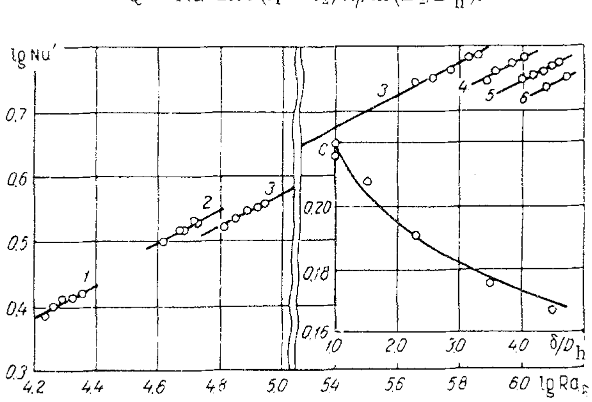

Fig. 3. Relation between the numbers Nu' and Ra_δ: 1) with δ/D_h = 0.5; 2) 0.93; 3) 1.5; 4) 2.36; 5) 3.5; 6) 4.5.

Because of the complexity of the phenomenon under examination, the value of the modified Nusselt number has been determined experimentally by a number of investigators [2, 4, 6–10]. A summary of the calculation relations that they obtained is given in the table.

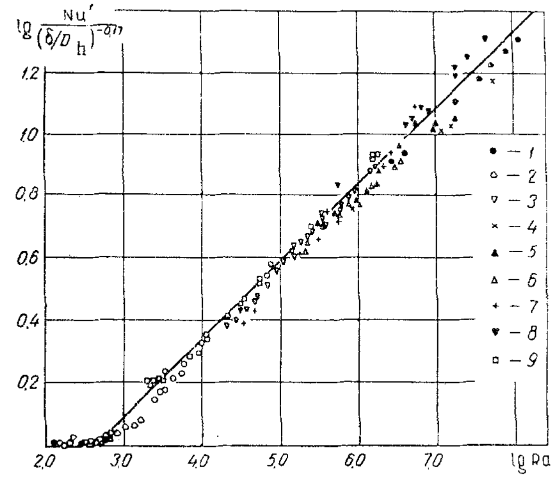

Fig. 4. Comparison of Eq. (1) with experimental data [2, 6, 7, 10], recalculated according to (1). 1, 2, 3) for air, H_2 and CO_2, respectively [6]; 4, 5, and 6) for water, and transformer and machine oil [7]; 7 and 8) for silicone and water $_2$10]; 9) for CO_2 [2].

The calculated Nu' number for the identical case of heat transfer through a cylindrical layer filled with air (D_2 = 0.16 m, D_h = 0.28 m, t_1 = 69° C, t_2 = 29° C, and the thermophysical properties determined according to [4] at the mean air temperature in the layer) are given in the last column of the table and show a measure of agreement which could be considered satisfactory.

The considerable possible maximum errors of each of these equations, as well as the divergence of the results of calculating the same case according to different equations, give reason for further investigations of the heat transfer through a layer.

It is also important to draw attention to the inconsistency that in the formulas of Kraussold, Mikheev, and Boyarintsev at $Ra_\delta \geq 10^6$ the exponent for Ra_δ is decreased, while it was established by the visual observations of Liu, Mueller, and Landis, for heat transfer in this region, that turbulence was excited and developed, and therefore such a decrease in the exponent is dubious.

We carried out tests on coaxial cylinders in the equipment whose general form was described above, along with the method of performing the tests.

The amount of heat transferred by radiation per 1 m length was calculated according to the relation

$$q_{rad} = \varepsilon_r C_s \pi D_h [(T_1/100)^4 - (T_2/100)^4].$$

In calculating the reduced emissivity of the system, we assumed that $\varepsilon_1 = \varepsilon_2 = 0.045$ [11].

Knowing the heat transferred by convection and conduction through the layer, $q = q_{el} - q_{rad}$, we can find the equivalent thermal conductivity:

$$\lambda_{eq} = q \ln (D_2/D_h) / \Delta t\, 2\pi.$$

From the values obtained for λ_{eq} and Δt, we calculated Nu' and Ra_δ, and the relation between them ex-

Relations for Calculating Heat Transfer in Horizontal Cylindrical Layers

Author	Relations proposed	Nu′
W. Beckmann	From the graphs of [6]	5.0
H. Kraussold	$(\mathrm{Gr}_\delta\,\mathrm{Pr}) \leqslant 10^3;\quad \mathrm{Nu}' = 1;$	
	$10^{3.8} < (\mathrm{Ra}_\delta)_f < 10^6;\quad \mathrm{Nu}' = 0.11\,(\mathrm{Ra}_\delta)_f^{0.29};$	5.6
	$(\mathrm{Ra}_\delta)_f > 10^6;\quad \mathrm{Nu}' = 0.40\,(\mathrm{Ra}_\delta)_f^{0.20}$	
H. Niemann	$(\mathrm{Ra}_\delta)_f \leqslant 10^8;\quad \mathrm{Nu}' = 1 + \dfrac{0.119\,(\mathrm{Ra}_\delta)_f^{1.27}}{(\mathrm{Ra}_\delta)_f + 1.45\cdot 10^4}$	5.5
D. I. Boyarintsev	$\left[\mathrm{Ra}_\delta\left(\dfrac{L_1}{L_h}\right)^3\right]_f \leqslant 10^3;\quad \mathrm{Nu}' = 1;$	
	$10^4 < \left[\mathrm{Ra}_\delta\left(\dfrac{L_1}{L_h}\right)^3\right]_f < 10^7;\quad \mathrm{Nu}' = 0.062\left[\mathrm{Ra}_\delta\left(\dfrac{L_1}{L_h}\right)^3\right]_f^{1/3};$	6.6
	$10^7 < \left[\mathrm{Ra}_\delta\left(\dfrac{L_1}{L_h}\right)^3\right]_f < 10^{10};\quad \mathrm{Nu}' = 0.22\left[\mathrm{Ra}_\delta\left(\dfrac{L_1}{L_h}\right)^3\right]_f^{1/4}$	
M. A. Mikheev	$(\mathrm{Ra}_\delta)_f \leqslant 10^3;\quad \mathrm{Nu}' = 1;$	
	$10^3 < (\mathrm{Ra}_\delta)_f < 10^6;\quad \mathrm{Nu}' = 0.105\,(\mathrm{Ra}_\delta)_f^{0.3}$	6.1
	$10^6 < (\mathrm{Ra}_\delta)_f < 10^{10};\quad \mathrm{Nu}' = 0.40\,(\mathrm{Ra}_\delta)_f^{0.2}$	
Chen-Ya Liu W. K. Mueller, F. Landis	$\left(\dfrac{\mathrm{Pr}^2\,\mathrm{Gr}_\delta}{1.36+\mathrm{Pr}}\right)_f < 10^3;\quad \mathrm{Nu}' = 1;$	4.3
	$10^{3.5} < \left(\dfrac{\mathrm{Pr}^2\,\mathrm{Gr}_\delta}{1.36+\mathrm{Pr}}\right)_f < 10^6;\quad \mathrm{Nu}' = 0.135\left(\dfrac{\mathrm{Pr}^2\,\mathrm{Gr}_\delta}{1.36+\mathrm{Pr}}\right)_f^{0.278}$	
J. Gargaud, B. Gasc	From the graphs of [2]	6.5

pressed in coordinates $\lg \mathrm{Nu}' - \lg \mathrm{Ra}_\delta$ (Fig. 3). It may be seen from the figure that, for each ratio δ/D_h, this relation is linear, and may therefore be expressed by an equation of the form

$$\lg \mathrm{Nu}' = \lg C + m \lg \mathrm{Ra}_\delta.$$

The slopes m and the constants C of the straight lines were found by the least squares method. For all the ratios δ/D_h, the value of m proved to be the same, and equal to 0.25. The dependence $C = f(\delta/D_h)$ (Fig. 3), following the least squares operation, was expressed by the equation

$$C = 0.22(\delta/D_h)^{-0.17} = 0.22(D_h/\delta)^{0.17};\ D_h/\delta = 1-0.22.$$

Thus, the parametric equation for heat transfer under free convection in a horizontal cylindrical symmetrical layer takes the form

$$\mathrm{Nu}' = 0.22(\mathrm{Ra}_\delta)_f^{0.25}(D_h/\delta)^{0.17},\ 4.2 \leq$$
$$\leq \lg(\mathrm{Ra}_\delta)_f \leq 6.2, \quad 0.22 \leq D_h/\delta \leq 1. \qquad (1)$$

The maximum error of (1) relative to the experimental data lies in the range ±2.5%.

It may be seen from Fig. 4 that the data of [2, 6, 7, 10] are described by the proposed equation with satisfactory accuracy.

The relation obtained permits calculation with satisfactory accuracy of heat transfer through gaseous and liquid cylindrical symmetrical layers in the range $3 \leq \lg(\mathrm{Ra}_\delta)_f \leq 8$.

It follows from Fig. 4 that in generalizing the experimental data of the different authors, allowing for the ratio δ/D_h, no clear variation is observed in the exponent of the Rayleigh number when its value is $(\mathrm{Ra}_\delta)_f \geq 10^6$ in the heat transfer equation, as was shown in [4, 7, 8].

NOTATION

Nu'—modified Nusselt number, $\mathrm{Nu}' = \lambda_{eq}/\lambda_f$; λ_{eq}—equivalent thermal conductivity; λ_f—thermal conductivity of liquid or gas; l—length of layer; D_h, r—diameter and radius of heater; D_2, R—diameter and radius of external cylinder; t_1 and t_2—temperature of internal and external cylinder, respectively; Ra_δ—Rayleigh number, $\mathrm{Ra}_\delta = (\mathrm{PrGr})_\delta$; Pr—Prandtl number; e—eccentricity; Gr_δ—Grashof number, $\mathrm{Gr}_\delta = g\delta^3\beta\Delta t/\nu^2$; δ—layer thickness; $\Delta t = t_1 - t_2$; β—volume expansion coefficient; ν—kinematic viscosity; g—acceleration due to gravity; ε_r—reduced emissivity of system; $\varepsilon_r = 1/[1/\varepsilon_1 + (D_h/D_2)(1/\varepsilon_2 - 1)]$; ε_1 and ε_2—emissivity of heater and external tube, respectively; Θ—dimensionless temperature, $\Theta = (t_1 - t_2)_e/(t_1 - t_2)_0$; $(t_1 - t_2)_e$ and $(t_1 - t_2)_0$—temperature difference of heat-source element when displaced from center, and when in the central position, respectively; η—dimensionless displacement.

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